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(1) Show that the mass and energy are equivalent according to special theory of relativity and obtain an expression between them.

Prove $E = mc^2$

In classical dynamics, Energy may be defined in term of work which is product of displacement and force. But by Newton's second law motion, the rate of change of momentum is called force, i.e

$$\frac{d}{dt}(mv) = F = \frac{dP}{dt}$$

But theory of relativity mass and velocity are variable.

$$\therefore F = m\frac{dv}{dt} + v\frac{dm}{dt}$$

$\therefore$ Kinetic energy,

$$dE_1 = F \cdot dx \quad \text{(By work-Energy theorem)}$$

$$= m\frac{dv}{dt} \cdot dx + v\frac{dm}{dt} \cdot dx$$

$$= m \cdot dv \cdot \frac{dx}{dt} + v \cdot dm \cdot \frac{dx}{dt} \qquad \because \frac{dx}{dt} = v$$

$$\therefore dE_1 = mv \cdot dv + v^2 \cdot dm \quad \text{—— (i)}$$

The relativistic law of variation the mass with velocity

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Squaring both side

$$m^2 = \frac{m_0^2}{1 - \frac{v^2}{c^2}} = \frac{m_0^2 c^2}{c^2 - v^2}$$

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$m^2(c^2 - v^2) = m_0^2 c^2$

$m^2c^2 - m^2v^2 = m_0^2 c^2$

differciatiating both side,

$\because$ m & v is variable, but m_0 and c (velocity of light) are constant

$\therefore c^2 \cdot 2m \cdot dm - v^2 \cdot 2m\, dm - m^2 2v \cdot dv = 0$

Dividing by 2m to both side, we get,

$c^2 \cdot dm - v^2 \cdot dm - mv \cdot dv = 0$

$\Rightarrow mv \cdot dv + v^2 \cdot dm = c^2 \cdot dm$

from eqn (i), we get,

$dE_1 = c^2 \cdot dm$ —— (ii)

When the body is accelerated from zero velocity to v, its mass increases from m_0 to m

So, total kinetic energy,

$$\int dE_1 = \int_{m_0}^{m} c^2 \cdot dm = c^2 \int_{m_0}^{m} dm = c^2 (m - m_0)$$

$\therefore E_1 = c^2 (m - m_0)$ —— (iii)

This relation shows that the K.E. of motion is the activating influence of the increase of mass and rest mass m_0 has been understood as an internal store of energy in the body. Since the total energy E possessed by the moving body is made of the K.E. of the motion and they stored up internal energy

$E = E_1 + m_0 c^2$

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$\therefore E = (m - m_0)c^2 + m_0c^2$ (from eqn. (iii))

$= mc^2 - \cancel{m_0c^2} + \cancel{m_0c^2}$

$\therefore \boxed{E = mc^2}$ Proved

This is famous Einstein's mass energy relation, which states a universal equivalence between mass and Energy.